You’ve probably heard the old saying “One picture is worth a thousand words.” Most people nod in agreement when this statement is made, without realizing just how powerful a picture, or a diagram, can be. (Note that words in bold type are terms that are defined in this book’s glossary.) A diagram has many advantages over verbal communication. For example, a diagram can show positional relationships far more easily and clearly than a verbal description can. To attempt to clarify ideas in their own minds, some people talk to themselves or to others about those ideas. Similarly, a diagram can help clarify ideas and solve problems that lend themselves to visual representations.

One of the best examples of a diagram in the professional world is a blueprint. An architect’s blueprint expresses ideas concisely in a visual form that leaves little to interpretation. Words are added only to indicate details that are not visually evident. A blueprint illustrates one of the strengths of diagrams: the ability to present the “whole picture” immediately.

Problem solving often revolves around information and how it is organized. When you draw a diagram, you organize information spatially, which then allows the visual part of your brain to become
more involved in the problem-solving process. In this chapter you will learn how diagrams can be used to clarify ideas and solve a variety of problems. You’ll improve your diagramming abilities, and you’ll discover that a diagram can help you understand and correctly interpret the information contained in a problem. You’ll also see the value of using diagrams as a problem-solving strategy.

Solve this problem by drawing a diagram.

MODERN BASKETBALL ASSOCIATION

A new basketball league was formed in which each of the teams will play three games against each of the other teams. There are seven teams: the Antelopes, the Bears, the Cubs, the Dusters, the Eagles, the Foxes, and the Goats. How many games will be played in all? Do this problem carefully before reading on.

As you read in Chapter 0, you’ll see many different problems as you work through this book. The problems are indicated by an icon of a dog. To get the maximum benefit from the book, solve each of the problems before reading on. You gain a lot by solving problems, even if your answers are incorrect. The process you use to solve each problem is what you should concentrate on.

You could use many different diagrams to solve the Modern Basketball Association problem. But you could also solve this problem
in ways that do not involve diagrams. As you read in Chapter 0, throughout this book you will see some of the same problems in different chapters and solve them with different strategies. This will help you become a better problem solver in two ways: by solving many different problems and by solving the same problem in many different ways. In this chapter, the solutions involve diagrams. If you solved the Modern Basketball Association problem without a diagram, try solving it again with a diagram before reading on.

What comes next is a solution process that is attributed to a student. The people mentioned in this book are real students who took a problem-solving class at either Luther Burbank High School in Sacramento, California, or at Sierra College in Rocklin, California. In those classes, the students presented their solutions on the board to their classmates. Ted Herr and Ken Johnson, the authors of this book, taught these classes. Our students presented their solutions because we felt that the other students would benefit greatly from seeing many different approaches to the same problem. We didn't judge each student's solution in any way. Rather, we asked each member of the class to examine each solution that was presented and decide which approach or approaches were valid or, perhaps, better.

We have tried to re-create the same learning atmosphere in this book. Sometimes you'll see several different approaches to a problem in this book, but for the most part those approaches and the resulting solutions won't be judged. You are encouraged to make up your own mind about the quality of the approaches. You may have been led to believe that there is always one right way and many wrong ways to solve problems. This notion couldn't be further from the truth. There are many right ways to solve problems, and you are encouraged to solve the problems in this book more than once, using different methods.

Here's how Rita solved the Modern Basketball Association problem:

She used a letter to represent each team. A stood for Antelopes, B stood for Bears, and so on, through G. She drew a diagram that showed the letters arranged in a circle. Next to each letter she drew a dot.
She then drew a line from A to B to represent the games played between the Antelopes and the Bears. Then she drew a line from A to C to represent the games played between the Antelopes and the Cubs.

She finished representing the Antelopes’ games by drawing lines from A to D, E, F, and G.

Next she drew the lines for the Bears. She’d already drawn a line from A to B to represent the games the Bears played against the Antelopes, so the first line she drew for the Bears was from B to C.

She continued drawing lines to represent the games that the Bears played against each other team. From C she drew lines only to D, E, F, and G because the lines from C to A and from C to B had already been drawn. She continued in this way, completing her diagram by drawing the lines needed to represent the games played by the rest of the teams in the league. (Note that when she finally got to the Goats, she did not need to draw any more lines because the games the Goats played against each other team had already been represented with a line.)
She then counted the lines she’d drawn. There were 21. She multiplied 21 by 3 (remember that each line represented three games), and she came up with an answer of 63 games. Finally, Rita made sure that she’d answered the question asked. The question was “How many games will be played in all?” Her answer, “Sixty-three games will be played,” accurately answers the question.

Mirka solved this problem with the diagram shown below. She also used the letters A, B, C, D, E, F, and G to represent the teams. She arranged the letters in a row and, as Rita did, she drew lines from team to team to represent games played. She started by drawing lines from A to the other letters, then from B to the other letters, and so on. She drew 21 lines, multiplied 21 by 3, and counted 63 games.

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**MODEL TRAIN**

Jenny’s model train is set up on a circular track. Six telephone poles are spaced evenly around the track. The engine of Jenny’s train takes 10 seconds to go from the first pole to the third pole. How long would it take the engine to go all the way around the track? Solve the problem carefully before reading on.

If you read the problem quickly and solved it in your head, you might think the answer is 20 seconds. After all, the problem states that the engine can go from the first pole to the third pole in 10 seconds, which is three poles out of six and apparently halfway around the track. So it would take the engine 2 times 10, or 20 seconds, to go all the way around the track. But this answer is wrong. The correct answer becomes apparent when we look at a diagram.
Dustin’s diagram is shown at right. Dustin explained that the train goes one-third of the way around the track in 10 seconds, not halfway around the track. So the train goes around the entire track in 3 times 10 seconds, or 30 seconds.

Phong drew the same diagram, but he interpreted it differently. He explained that if it takes 10 seconds to go from the first pole to the third pole, then it takes 5 seconds to go from the first pole to the second pole. So, it takes 5 seconds to go from pole to pole. There are six poles, so it takes the train 30 seconds to go all the way around the track.

Pete interpreted the problem as Phong did, but he didn’t draw a diagram. Thus, he neglected the fact that the train must return from the sixth pole to the first pole in order to travel all the way around the track. Therefore, he got the incorrect answer of 25 seconds.

The diagram helped Dustin and Phong solve the Model Train problem. If you used a diagram to solve the problem, you probably got the correct solution. If you were able to get the correct solution without drawing a diagram, think back on your process. You probably visualized the train track in your mind, so even though you didn’t actually draw a diagram, you could “see” a picture.

Do you get the picture? Do you see why diagrams are important? Research shows that most good problem solvers draw diagrams for almost every problem they solve. Do not resist drawing a diagram because you think you can’t draw, or that smart people use only equations to solve problems, or whatever. Just draw it!

ALIEN INVADERS

Sam, Mamie, Ralph, and Gail are all skilled at the video game Alien Invaders. Gail consistently scores higher than Ralph. Sam is better than all of them, and Mamie is better than Ralph. Who is the better player, Gail or Mamie? Before reading on, use the clues in the problem to solve it.
Jamie drew the diagram shown below. Each figure’s height represents that player’s skill level: the tall figures are more skilled than the short figures. (Jamie is very tall, and she thinks that tall people are good at video games.) As the diagram shows, Sam is the best player, followed by Mamie, Gail, and Ralph. Thus, Mamie is better than Gail.

![Diagram showing player heights]

Kurt drew the diagram shown below. The larger heads represent the players with more video game prowess, and the smaller heads represent the players with less skill. (Kurt has a very large head, so he thinks that large-headed people are good at video games.) As the diagram shows, Sam is the best, followed by Gail, Mamie, and Ralph. So Kurt’s answer is that Gail is better than Mamie.

![Diagram showing head sizes]

Rena drew the diagram shown at right. Her diagram shows arrows that represent each relationship described in the problem. The arrows point from one person to another, and they show that the person the arrow starts from is better at playing Alien Invaders than the person the arrow points to. When no arrow is shown, then the problem did not describe a relationship between them. Rena’s diagram makes it clear that the question can’t be answered because there is no way of establishing a comparison between Mamie and Gail.
Curly used a shovel to dig his own swimming pool. He figured he needed a pool because digging it was hard work and he could use it to cool off after working on it all day. He also planned to build a rectangular concrete deck around the pool that would be 6 feet wide at all points. The pool is rectangular and measures 14 feet by 40 feet. What is the area of the deck?

As usual, solve this problem before continuing.

Simon drew the following diagram to show the correct dimensions of the deck and pool, which together are 12 feet longer and 12 feet wider than the pool alone.

The diagram helps show the difficult parts of the problem. However, Simon solved the problem incorrectly by finding the outside perimeter of the pool and deck together, then multiplying the perimeter by the width of the deck.

\[52 \text{ feet} + 26 \text{ feet} + 52 \text{ feet} + 26 \text{ feet} = 156 \text{ feet}\]
\[156 \text{ feet} \times 6 \text{ feet} = 936 \text{ square feet}\]

His approach overcounts the corners.
Rajesh used the same diagram, but he solved the problem by first computing the area of the deck along the sides of the pool, then adding in the corners of the deck.

Two lengths: 40 ft × 6 ft × 2 = 480 sq ft
Two widths: 14 ft × 6 ft × 2 = 168 sq ft
Four corners: 6 ft × 6 ft × 4 = 144 sq ft
Total = 792 sq ft

May’s diagram shows the corners attached to the length of the deck.

She calculated the area as follows:

52 ft × 6 ft = 312 sq ft
312 sq ft × 2 = 624 sq ft for extended lengths
14 ft × 6 ft = 84 sq ft
84 sq ft × 2 = 168 sq ft for widths
Total = 624 sq ft + 168 sq ft = 792 sq ft
Hung solved this by first computing the area of the pool and the deck together, then subtracting the area of the pool, leaving the area of the deck.

Area of entire figure = 52 ft × 26 ft = 1352 sq ft
Area of pool alone = 40 ft × 14 ft = 560 sq ft
Area of deck = 1352 ft - 560 ft = 792 sq ft

Farmer Ben has only ducks and cows. He can’t remember how many of each he has, but he doesn’t need to remember because he knows he has 22 animals and that 22 is also his age. He also knows that the animals have a total of 56 legs, because 56 is also his father’s age. Assuming that each animal has all legs intact and no more, how many of each animal does Farmer Ben have? Do this problem, then read on.

Bill drew the following diagram:

“These 22 circles represent the 22 animals. First, I made all of the animals into ducks.” (Bill is not much of an artist, so you just have to believe that these are ducks.) “I gave each animal two legs because ducks have two legs.”
Then I converted the ducks into cows by drawing extra legs. The ducks alone had 44 of the 56 legs initially, so I drew 12 more legs (six pairs) on 6 ducks to turn them into cows. So there are 6 cows and 16 ducks.

Of course, Ben might have a problem when his father turns 57 next year.

Any idea that can be represented with a picture can be communicated more effectively with that picture. By showing what a person is thinking, a diagram becomes a problem-solving strategy. A diagram clarifies ideas and communicates those ideas to anyone who looks at it. Many people use diagrams as a part of their jobs, especially those that require a planning stage to complete a project. Occupational diagrams include blueprints, project flow charts, and visual representations of concepts. And diagrams are often used to show position and direction because these concepts can be communicated more easily and clearly with a diagram than with words.

One final word about diagrams: Had the person who coined the phrase “One picture is worth a thousand words” lived in modern times, he undoubtedly would have said, “One TV is worth a thousand radios.” Today we recognize that a television image gives clearer and far more information than a radio’s verbal description. For these reasons, a diagram works to solve problems better and to communicate more effectively.
Problem Set A

You must draw a diagram to solve each problem.

1. **WORM JOURNEY**

   A worm is at the bottom of a 12-foot wall. Every day the worm crawls up 3 feet, but at night it slips down 2 feet. How many days does it take the worm to get to the top of the wall?

2. **UPS AND DOWNS OF SHOPPING**

   Roberto is shopping in a large department store with many floors. He enters the store on the middle floor from a skyway, and he immediately goes to the credit department. After making sure his credit is good, he goes up three floors to the housewares department. Then he goes down five floors to the children’s department. Then he goes up six floors to the TV department. Finally, he goes down ten floors to the
main entrance of the store, which is on the first floor, and leaves to go to another store down the street. How many floors does the department store have?

3. FOLLOW THE BOUNCING BALL
A ball rebounds one-half the height from which it is dropped. The ball is dropped from a height of 160 feet and keeps on bouncing. What is the total vertical distance the ball will travel from the moment it is dropped to the moment it hits the floor for the fifth time?

4. FLOOR TILES
How many 9-inch-square floor tiles are needed to cover a rectangular floor that measures 12 feet by 15 feet?

5. COUNTING ON NINJA TURTLES
Joanne sets up her Teenage Mutant Ninja Turtles in a big circle, spacing each turtle at an equal distance from its neighbors. She then begins counting them in order around the circle, but she loses track of where she started before she finishes counting. Then she notices that the sixth turtle is directly opposite the seventeenth turtle, and she realizes that she can still figure out how many turtles are in the circle. How many turtles are in the circle?

6. DANGEROUS MANEUVERS
Somewhere in the Mojave Desert, the army set up training camps named Arachnid, Feline, Canine, Lupine, Bovine, and Thirty-Nine. Arachnid is 15 miles from Canine. Bovine is 12 miles from Lupine. Feline is 6 miles from Thirty-Nine. Lupine is 3 miles from Canine. Bovine is 9 miles from Thirty-Nine. Bovine is 7 miles from Canine. Thirty-Nine is 1 mile from Arachnid. Feline is 11 miles from Lupine. No other pairs of training camps are connected by roads.
Answer each of the following questions (in each answer, indicate both the mileage and the route): What is the shortest route from

- Feline to Bovine?
- Lupine to Thirty-Nine?
- Canine to Feline?
- Arachnid to Lupine?
- Canine to Thirty-Nine?
- Lupine to Bovine?
- Arachnid to Feline?
7. **RACE**

Betty, Cathy, Isabel, Lani, Alma, and Ursula ran an 800-meter race. Alma beat Isabel by 7 meters. Ursula beat Betty by 12 meters. Alma finished 5 meters ahead of Lani but 3 meters behind Ursula. Cathy finished halfway between the first and last girls. In what order did the girls finish? What were the distances between each girl?

8. **A WHOLE LOTTA SHAKIN’ GOIN’ ON!**

If six people met at a party and all shook hands with one another, how many handshakes would be exchanged?

9. **HAYWIRE**

A telephone system in a major manufacturing company has gone haywire. The system will complete certain calls only over certain sets of wires. So, to get a message to someone, an employee of the company first has to call another employee to start a message on a route to the person the call is for. As far as the company can determine, these are the connections:

- Cherlondia can call Al and Shirley (this means that Cherlondia can call them, but neither Al nor Shirley can call Cherlondia). Al can call Max.
- Wolfgang can call Darlene, and Darlene can call Wolfgang back.
- Sylvia can call Dalamatia and Henry. Max can get calls only from Al.
- Carla can call Sylvia and Cherlondia. Shirley can call Darlene.
- Max can call Henry. Darlene can call Sylvia. Henry can call Carla.
- Cherlondia can call Dalamatia.

How would you route a message from

- Cherlondia to Darlene?
- Shirley to Henry?
- Carla to Max?
- Max to Dalamatia?
- Sylvia to Wolfgang?
- Cherlondia to Sylvia?
- Henry to Wolfgang?
- Dalamatia to Henry?
10. CONNECTIONS

How have you used diagrams in other classes?

11. WRITE YOUR OWN PROBLEM

In each chapter you'll be given the opportunity to write your own problem that can be solved by using the strategy you studied in that chapter. The book will give you suggestions for how to go about writing these problems. Each time you write your own problem, solve it yourself to be sure that it's solvable. Then give it to another student to solve and, as needed, also to help you with the problem's wording.

Create your own Draw a Diagram problem. Model it after either this chapter’s Worm Journey problem or the Ups and Downs of Shopping problem.