Chapter 6 – Polynomials & Polynomial Functions

6.1 Using Properties of Exponents

a^n – a × a × a × a …. n times

Check out the rule sheet for exponents and scientific notation

- For $y_1 = x$ y_1 = 1,2,3,4,5,6… as x goes from 1 to ∞
- For $y_2 = x^2$ y_2 = 1,4,9,16,25,36…as x goes from 1 to ∞
- Given the 2 equations above, if you double x, how does y change?

  $y_1 = 2x$ (just doubles)

  y_2 = ?

Try: if the volume of a sphere is modeled by: $\frac{4}{3}\pi r^3$ what does the formula look like if you double the radius? How much bigger is the volume when you double the radius?

Practice: Pg. 326 2,10,13,17,27,33,45,48,50,53

6.2 Evaluating & Graphing Polynomial Functions

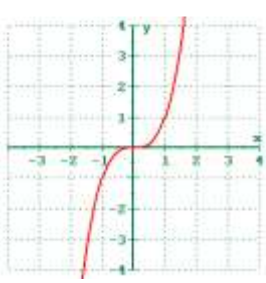
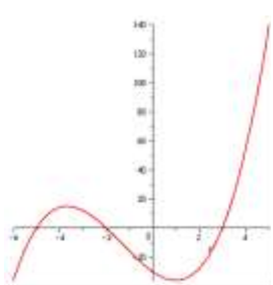
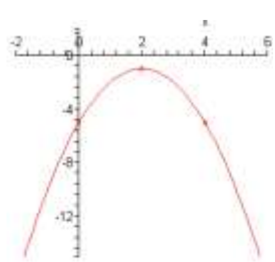
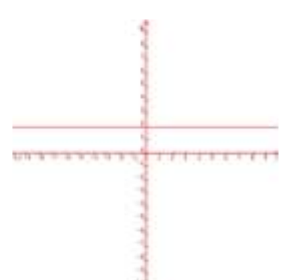
$F(x) = 4x^2 - 7x^5 + 2x^2 - 4 + 6x^3$ ←standard form

<u>Leading coefficient</u> – constant in front of the variable with the highest exponent. Above its ____? <u>Constant term</u> – term with n^0 or no variable

1) Classify functions by "degree" (highest)
   - Linear function – highest power is x^1
   - Quadratic function – highest power is x^2
   - Cubic function – highest power is x^3
   - Quartic function – highest power is x^4
2) Classify by # of terms
   - Constant – 1 term with no variable
   - Binomial – 2 terms; $2x + 1$ or $x^2 - 3$
   - Trinomial – 3 terms $x^2 + 3x - 4$

"Solving" Quadratics –

1) Try direct substitution –
2) Synthetic Division or "substitution" – "add down, multiply up"
3) Look at end behavior - the function will behave (at large values of x) like its highest power

Group: Pg. 333 15-26,49-52
Practice: pg. 333 4-5,31,35,37,41,45,47,65,69

6.3 Adding, Subtracting & Multiplying Polynomials
2 ways – horizontal or vertical add/subtract/multiply

Practice: Pg. 341 2,12,17,21,29,33,44,62,72

6.4 Factoring & Solving Polynomial Equations
Common factoring: $x^2 - x - 12 = (x+3)(x-4)$
Perfect square: $x^2 + 10x + 25 = (x+5)^2$
Diff of 2 squares: $x^2 - 16 = (x+4)(x-4)$
Monomial factors: $5x^2 + 15x = 5x(x+3)$

Cubes: $a^3+b^3 = (a+b)(a^2-ab+b^2)$
$a^3-b^3 = (a-b)(a^2+ab+b^2)$
Grouping:
Quadratic factoring – replacing x^2 with x (used when you have squares with even exponents) $81x^4 - 16 = (9x^2)^2 - (4)^2$ = difference of 2 squares
$(9x^2-4)(9x^2+4)$ = another diff of 2 squares
$(9x^2+4)(3x-4)(3x+4)$
The factors, set = 0 are called solutions, roots, zeros or x-intercepts

Group: Pg. 348 28-32 evens
Practice: Pg. 348 3,6,19,27-31,33,45,51,59,60,71,73,77,79,81,85

6.5 The Remainder & Factor Theorems
Polynomial division can by done normally or by synthetic division
Zeros – are the factors that divide into the polynomial with no remainder

Practice: Pg. 356 5,9,13,15,23,27,31,39,43,47,51,55

6.6 Finding Rational Zeros
Rational zero theorem: $f(x) = 64x^3 + 120x^2 - 34x - 105$
has factors: $\frac{-3}{2}$, $\frac{-5}{4}$, $\frac{7}{8}$ → Numerators all factors of 105 (p)
→ Denominators all factors of 64 (q)

- factors will be in the form $\frac{p}{q}$

Practice: Pg. 362 1,5,11,15,19,23,29,39,43,45,53

6.7 Using the Fundamental Theorem of Algebra

Multiplicity – how many times the factor can be taken out of the polynomial
x^{odd} – is guaranteed one <u>real</u> root (zero)
x^n – has n total roots (including real and complex/imaginary)
Complex/imaginary roots always come in pairs (think of the quadratic formula with $\pm$ answers)
***Find the real roots first. You should have a quadratic left to factor.
Roots of: 4,-2 would yield factors of (x-4) and (x+2)
3i, -3i would yield factors of (x-3i) and (x+3i)
- what quadratic has the factors 3i,-3i???
(x-3i)(x+3i) Foil it and find out!!

$i = \sqrt{-1}$
$i^2 = -1$

Practice: Pg. 369 1,3,5,9,17,19,23,29,41,45,57

6.8 Analyzing Graphs of Polynomial Functions

Zeros of a polynomial are where the graph crosses or touches the x-axis (except imaginary)

- If multiplicity is even, the function crosses the x-axis
- If multiplicity is odd, the function touches and turns

Turning points – local max or local min

- x^n has n-1 turning points
- If function has n zeros, then it has exactly n-1 turning points

Groups: with graphing calculator pg. 376 8-11,29-34
Practice: Pg. 376 1-3,7,13,17,19,23,24,26

6.9 Modeling with Polynomial Functions

a = constant in front of the polynomial: $f(x) = a(x+3)(x-2)(x-5)$
to find a, plug in an (x,y) point on the function
direction of function – watch the sign on a, but also plug in the zeros everywhere except where it would make the function go to zero:

$f(x) = 2(x+3)(x-2)(x-5)$ → zeros are -3,2,5
$2x^3$ will be a normal x^3 (down on left, up on right)
$2(-3-2)(-3-5) = 2(-5)(-8) = 80$ ← line through -3 will be + slope
$2(2+3)(2-5) = 2(5)(-3) = -30$ ← line through 2 will be – slope
$2(5+3)(5-2) = 2(8)(3) = 40$ ← line through 5 will be + slope

Practice: Pg. 383 2,5,7,9,14-17,23,29